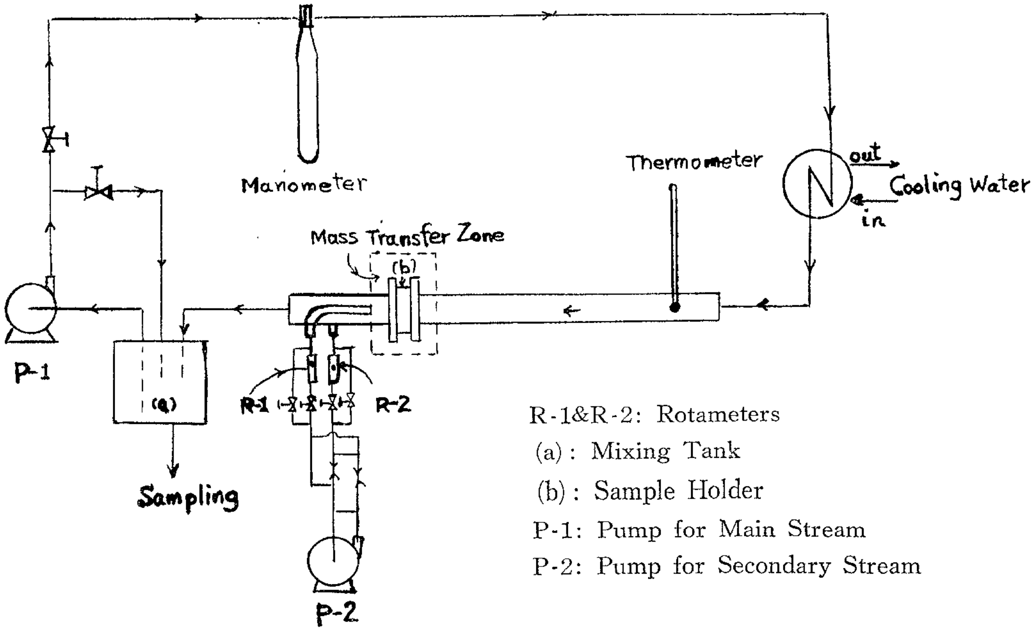
MASS TRANSFER COEFFICIENT AND VELOCITY PROFILE

Effects of Secendary Parallel Flow at Center of Main Flow on Mass Transfer Rate

Chen Shih-hsueh

I. Introduction

A study of slow mass trasfer rate from circular pipe to a steady flew fluid was reported in the Tunghai Journal (1) Vol. VIII, No. 1, pp 207-214 Jan. 1967. It is found that the velocity profile at the mass transfer zone is one of the most important factors which affected the mass transfer rate or the mass transfer ceefficients K_L of the system. In the experiments, the mass transfer zone is kept short compared with the diameter $(L/D<\frac{1}{2})$ then the concentration boundary layer thickness $\delta_{c\ max.}$ will be very thin that δ_c/D will be very very small. In the situation that $\delta_{c\ max.} \leq y|_{y^+=5}$ holds, then the velocity gradient inside the concentration boundary layer will be constant approximately (2). This means that one may correlate the mass transfer coeffficient k_L as a function of velocity gradient at wall because the velocity distribution inside the region will be a linear function of the velocity gradient at wall. A theoretical analysis for correlating the K_L



R-1&R-2: Rotameters
(a): Mixing Tank
(b): Sample Holder
P-1: Pump for Main Stream
P-2: Pump for Secondary Stream

Figure-1

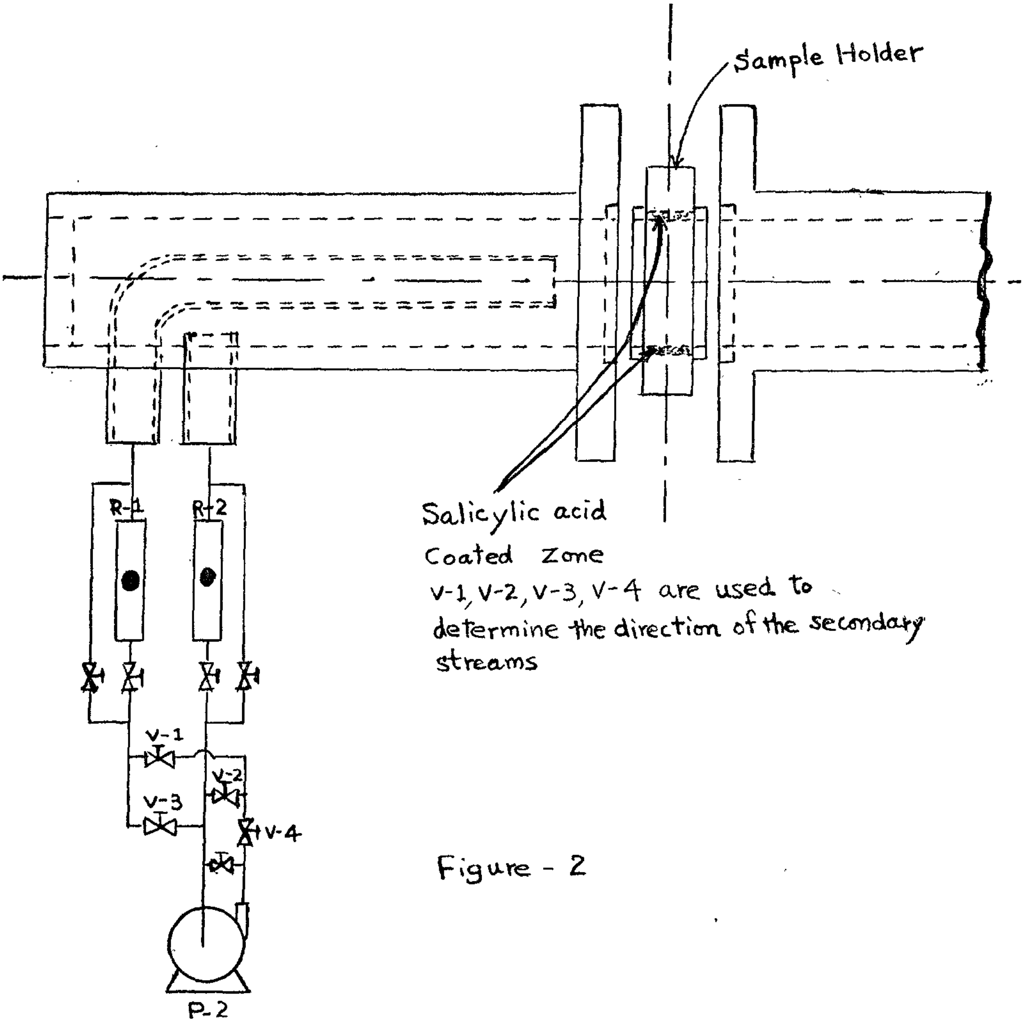
and the velocity gradient at wall of the system will be given in the next section. This experiment was designed to show (1) experimental verification of the correlation of k_L vs. velocity gradient at wall; (2) ways to change or contral the mass transfer rate without changing the main flow rate by introducing a secondary flow parallel to the main flow at center. The results shows good predictions of the effects of the seondary flow on the mass transfer rate and gives a way to change or control the transfer rate by not changing the flow rate of the main stream.

II. Apparatus and Theory

(1) Apparatus and Assumption

The apparatus was designed and constructed to permit formation of countercurrent or co-current parallel flow at center of the main stream. Figure-1 is the flow diagram of the system, and figure-2 is the details of the mass transfer zone and also showing how the secondary flows are made.

In figure-1, p-1 is the main pump. Over 90% of the output of p-1 is recycled to mixing tank



V-1, V-2, V-3, V-4 are used to determine the direction of the secondary streams

Figure - 2

(a), because the recycle stream is very strong in comparison with the tank hold-up, it is assumed that inside the tank, the acid concentration is homogeneous throughout. The secondary streams are made by pump p-2, and the flow directions are determined by the valves v-1 to v-4.

In mass transfer zone, the wall is coated with salicylic acid (which is only sparingly soluble in water). The whole wall is connected smoothly so that we can assume no roughness is introduced. For this system the following assumptions will be reasonable:

i) Hydraulic entrance and exit are long enough that a steady developed velocity profile is made at the mass transfer zone.

ii) δ_c max. $\leq y|_{y^+=5}$ that the velocity gradient inside the concentration boundary layer may regard as a constant.

iii) Surface concentration of salicylic acid in water at mass transfer zone is its saturated concentration.

iv) Mass flux of salicylic acid is very small, (this is true under these experimental conditions) that the flux neither changed the velocity profile nor changes that physical properties of the fluid.

v) Neglect the roughness introduced by dissolving out of the coated salicylic acid. (In this experiment, the quantity which dissloved are kept under 100 mg.)

(2) Correlation of k_L with velocity gradient at wall:

The fundamental equation for the system is:

$$V_z \frac{\partial C_A}{\partial z} = D_{AB}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r}\right)\right] \cdots\cdots (1)$$

Let

$$P = \text{const.} = -\left(\frac{dV_z}{dr}\right)_{r=R} = \left(\frac{dV_z}{dy}\right)_{y=0} = \left(\frac{dV_z}{dy}\right)_{y=\delta c} \cdots\cdots (2)$$

where $y = R - r \cdots\cdots (3)$

then $V_Z = Py \cdots\cdots (4)$

inside the mass diffusion region. It is assumed that the "similar" solution (3) exists as in the preceeding report, then as before with appropriate boundary conditions lead to

$$\eta = \frac{R-r}{\delta_c(z)} = \frac{y}{\delta_c(z)} \cdots\cdots (5)$$

$$C_A^* = \frac{C_{As}-C_A}{C_{As}-C_{A0}} = C_A^*(\eta) = 2\eta - 2\eta^3 + \eta^4 \cdots\cdots (6)$$

Then the eq. (1) may be reduced to

$$-P\delta_c\eta\left(\frac{\eta}{\delta_c}\right)\left(\frac{d\delta_c}{dz}\right)\left(\frac{dC_A^*}{d\eta}\right) = D_{AB}\left[\frac{1}{\delta_c^2}\left(\frac{d^2C_A^*}{d\eta^2}\right) - \frac{1}{(R-\delta_c\eta)}\frac{1}{\delta_c}\left(\frac{dC_A^*}{d\eta}\right]\right. \cdots\cdots (7)$$

Integrate with respect to η from 0-1 we get

$$-\int_0^1 P\eta^2\delta_c^2(R-\delta_c\eta)\left(\frac{d\delta_c}{dz}\right)\left(\frac{dC_A^*}{d\eta}\right)d\eta = D_{AB}\int_0^1]\,(R-\delta_c\eta)\left(\frac{d^2C_A^*}{d\eta^2}\right) - \delta_c\left(\frac{dC_A^*}{d\eta}\right)]d\eta \cdots (8)$$

$$\text{or } P\delta_c^2\left(\frac{d\delta_c}{dz}\right)\left(\frac{2}{15}R - \frac{1}{14}\delta_c\right) = 2RD_{AB} \cdots\cdots (9)$$

Integrate with respect to δ_c from $O-\delta_c$ and z from $O-L$ again get

$$\frac{1}{45}PD\delta_c^3 - \frac{1}{56}\delta_c^4 = D_{AB}DL \cdots\cdots (10)$$

or

$$\frac{1}{45}\left(\frac{\delta_c}{D}\right)^3-\frac{1}{56}\left(\frac{\delta_c}{D}\right)^4=\frac{D_{AB}L}{PD^3} \qquad (11)$$

$\left(\frac{\delta_c}{D}\right)$ is very small in the experimental conditions, then neglected the 4th. power term and get

$$\frac{\delta_c}{D}=45^{\frac{1}{3}}\left(\frac{D_{AB}L}{D^3}\right)^{\frac{1}{3}}P^{-\frac{1}{3}} \qquad (12)$$

or

$$\delta_c=3.554\,D_{AB}^{\frac{1}{3}}L^{\frac{1}{3}}P^{-\frac{1}{3}} \qquad (13)$$

$$\delta_c=\frac{1}{L}\int_0^L\delta_c dz=2.66D_{AB}^{\frac{1}{3}}L^{\frac{1}{3}}P^{-\frac{1}{3}} \qquad (14)$$

$$K_L=\frac{D_{AB}}{\delta_c}=0.375D_{AB}^{\frac{2}{3}}L^{-\frac{1}{3}}P^{\frac{1}{3}} \qquad (15)$$

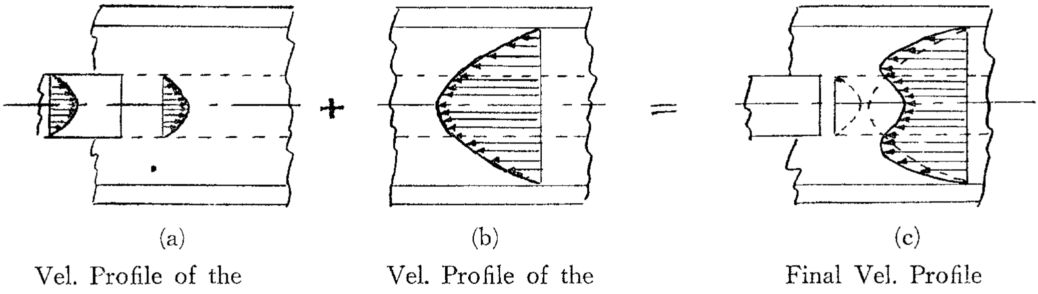
For a given condition, the D_{AB} & L are constant, so lead to the conclusion that

$$K_c\propto P^{\frac{1}{3}} \qquad (16)$$

(3) Qualitative Prediction of the Effects of a Secondary Parallel Flow at Center on Mass Transfer Rate.

3-1) Effects of Counter-current Secondary Stream on Mass Transfer Rate.

When going upstream, the secondary flow will die out by internal friction of the fluid. Since the used fluid water has very low viscosity and the mass transfer zone is very closed to the outlet of the secondary stream, it is assumed that the secondary flow keeps the velocity profile (as in the small pipe) at the mass transfer zone. The final velocity profile at the mass transfer zone may be as following:



(a) Vel. Profile of the Secondary Stream

(b) Vel. Profile of the Main Stream

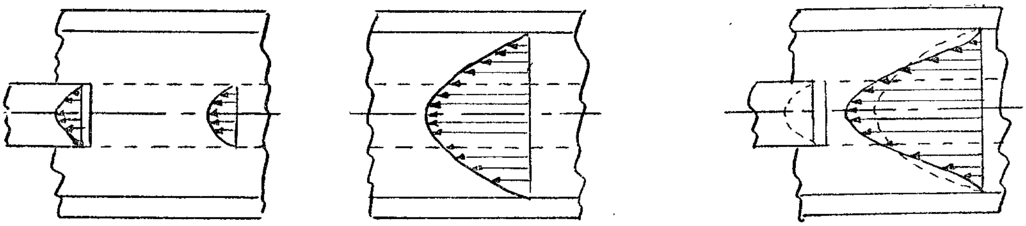
(c) Final Vel. Profile by Two Stream

Figure-3

Since the flow is kept steady, so mass or volume flow rate at any cross-section has to keep constant. Then counter-current secondary flow at center will increase the mass or volume flow rate near the wall in order to satisfy the continuity requirement. We may conclude that a counter-current secondary flow at center always increase the flow rate near the wall or we may say the velocity gradient near the wall will always be increased by introducing a counter-current secondary flow at center. The result will be more evident for small Re and strong secondary flow (large q/Q), and will be less evident for large Re and week secondary flow. Accoring to the preceeding analysis, increase the velocity gradient at wall will increase the K_L or mass transfer rate. Larger the q/Q, increase in k_L or mass transfer rate will be more evident.

3-2) Effects of Co-current Secondary Stream at Center on Mass Transfer Rate:

As in the case of counter-current stream, the final velocity profile at the mass transfer zone may be as following:



(a)
Vel. Profile of the
Secondary Stream

(b)
Vel. Profile of the
Main Stream

(c)
Final Vel. Profile
by Two Stream

Figure-4

Since the flow is kept steady, so mass or volume flow rate at any cross-section have to keep constant. Then co-current secondary flow at center will decrease the mass or volume flow rate mear the wall in order to satisfy the continuity requirement. We may conclude that a co-current secondary flow at center will decrease the mass or volume flow rate near the wall in order to satisfy the continuity requirement. We may conclude that a co-current secondary flow at cente will decrease the flow rate near the wall or we may say the velocity gradient near the wall will be decreased by introducing a secondary flow at center.

(4) Methed of Experiment

Salicylic acid is coated in the groove of sample holders and made smooth with respect to the pipe inside diameter. A small quantity of sample water is withdrawn at 5 to 10 min. intervals, and the concentration of salicylic acid in water is determined by a spectrophotometer (Color developer is 2 drops of $FeCl_3$ soln.) In the experiment, maximum acid concentration is about 35 mg. per liter. In this range, absorbance at 525 mu shows very good linearity with acid concentration. Concentration vs. time plot are made and k_L are calculated.

III. Results

(1) Effects of Counter-current Secondary Flow at Center on Mass Transfer Rate.

According to the preceeding analysis, the secondary flow will increase the mass transfer rate (or k_L). Figure-5 shows the results obtained at 28°C±2°C for Re=900 to Re=2700. From the figure we can conclude that k_L of the system do increase with increase in secondary flow rate.

Let

q=volume flow rate of the secondary stream

Q=volume flow rate of the main stream

$k_{L0}=k_L$ value for $q=0$,

plot the value $\triangle k_L=k_L-k_{L0}$ (for given Re) vs q/Q, we get the results as shown in figure-6 From this we may conclude that increase of the k_L value owing to the secondary flow depend on q/Q and independ on the Re of the main flow.

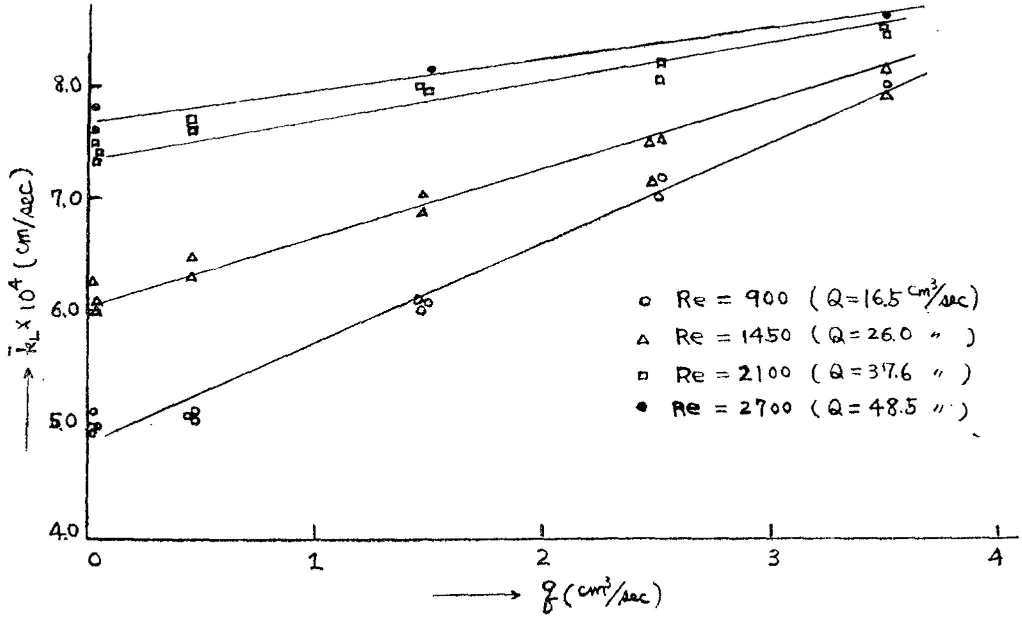


Figure-5

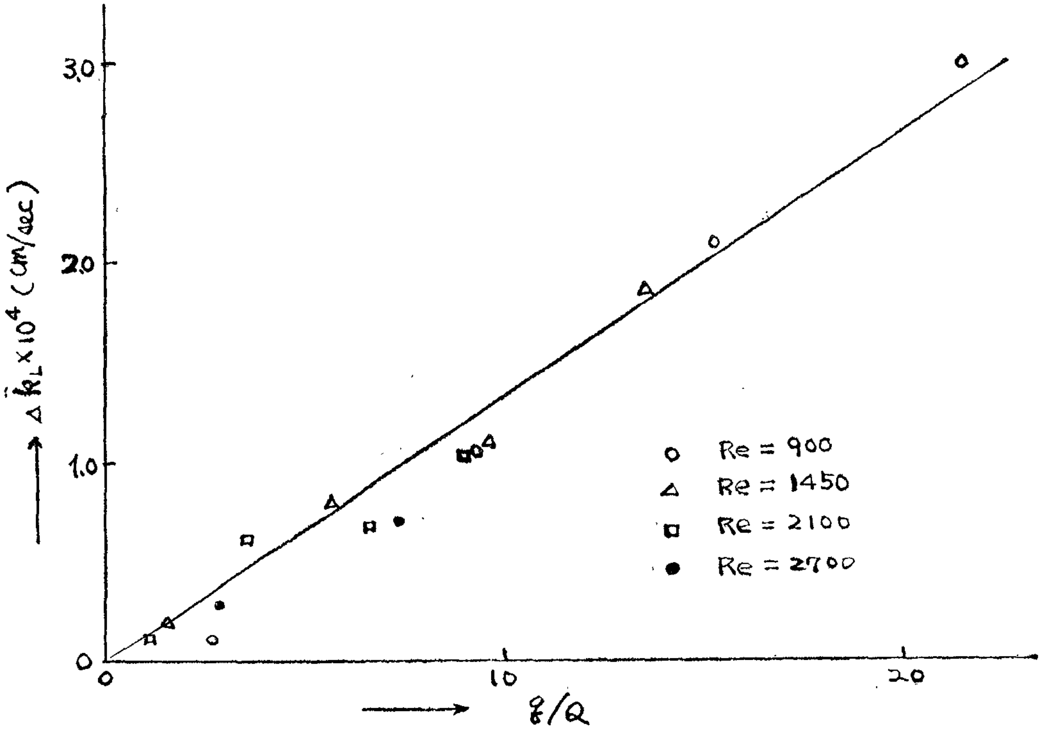


Figure-6

(2) Effects of Co-curent Secondary Flow at Center on Mass Transfer Coefficient:

According to the preceeding analysis, the secondary flow will decrease the mass transfer rate (for moderate flow rate). Figure-7 shows the results obtained at 28°C±2°C for Re=900 to Re=2700.

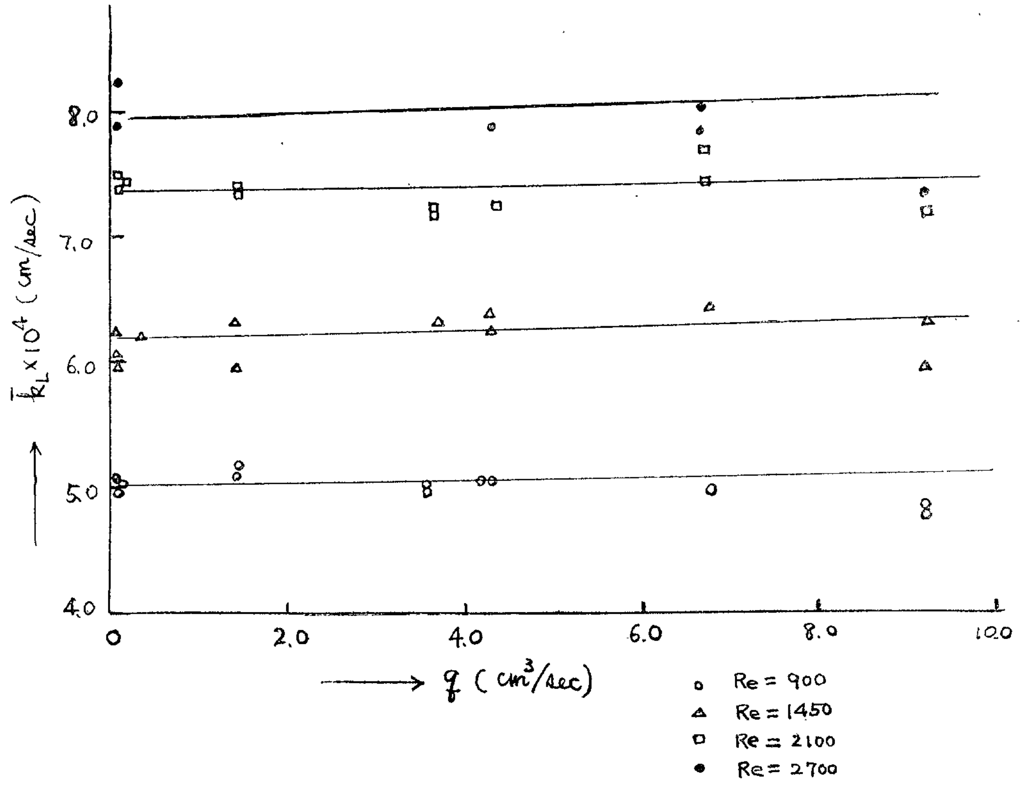


Figure-7

The effects were not evident in the experiment region.

IV. Conclusion

1) It is reasoned that a counter-current secondary stream at center of main stream will increase the velocity gradient at wall can't be proved directly because the apparatus used is not large enough for measuring the local velocity. But from the results that the k_L values are increased by introducing the countercurrent secondary stream at center tells us that the reasoning is corrected.

2) By the reasoning, a co-current secondary stream at center of the main stream will decrease the k_L values, but the results show that k_L's are almost kept constant for given Re. This may be counted for the turbulence introduced by the introduced secondary stream. The induced turbulence will always increase the velocity gradient at wall, so will increase the k_L. This is just compensated by the decreasing effects of the co-current secondary stream as results shown.

3) In the situation one want to increase the k_L value without changing the main flow rate, induced turbulence and introducing counter-current secondary flow both do the job. Induced turbulence method is very hard to be controlled but the second method, $\triangle k_L$ are proportional to q/Q, in which q may be adjusted easily. So introducing a counter-current secondary flow is a method

for increaasing the k_L's of the system with benefit that $\triangle k_L$'s are controllable.

NOMENCLATURE

A=mass transfer area

C_A=concentration of species A, C_{A0}=initial conc., C_{As}=saturated conc.,

C_A=local bulk conc., C^*_A=dimensionless conc.

D=diameter

D_{AB}=diffusivity of A in B

k_L, $\overline{k_L}$=local and mean mass transfer coefflcients

L=distance related to mass transfer area

q=volume flow rate of the secondary stream

Q=volume flow rate of the main stream

R=radius

P=velocity gradient at wall

t=time

v_z=axial velocity component of the main stream, v^*=friction velocity

v^+=dimensionless velocity

x, y, z,r,=coorinates, y^+=dimensienless coordinate, $(y^+=\frac{yv^*\rho}{\mu})$ η=dimensionless coordinate $(\eta=\frac{R-r}{\delta_c})$

W_A=rate of A dissolved

Re=Reynolds number

ρ=density, ν=kinematic viscosity

δ_c=concentration boundary layer thickness

μ=viscosity

References:

(1) Tungnai Journal, Vol. VII, No. 1, pp 207-214, Jan. 1967

(2) R. Byron Bird, Warren E. Stewart, Edwin N. Lightfoot; Transport Phenomena, pp-163, 1960

(3) Schlichting; Boundary Layer Theory, pp130, pp229.

Acknowledgements

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物質傳送係數速度分布之關係

於物質傳送部主流之中央，引進平行於主流的二次流對物質傳送速度之影響

陳世學

從圓管管壁向管內流動中之流體之物質傳送速度，可以由在主流中央部分引進平行而相對於主流方向之二次流使其增加。(亦就是增加物質傳送係數 $\bar{k}_L$。) 已被衆知的增加 $\bar{k}_L$ 之方法有增加系的渦流度，以及增加流速等。如在不欲增加系之流量之情形，多由增加系的渦流度使 $\bar{k}_L$ 增加。但用此法 $\bar{k}_L$ 之增加量不能控制，如引進平行而相對於主流的二次流，可增加 $\bar{k}_L$ 且其增加率爲二次流與主流體積流量之比值 (q/Q) 之線型函數。因 q 爲極易控制之量。是故在不改變主流流量下欲有限度地改變 $\bar{k}_L$ 值引進二次流爲一簡單而有効的方法之一。

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Mass transfer rate from circular pipe wall to steady flow fluids may be increased by increasing the flow rate or by introducing turbulence in the flow, etc. In the situation one want to increase the $\bar{k}_L$ value without changing the main flow rate, introducing turbulence will do the job but it is very hard to control the amount of $\bar{k}_L$ increase. By introducing counter-current secondry flow at center of main flow will increase the $\bar{k}_L$ without changing the main flow rate. The $\triangle\bar{k}_L$ are proportional to q/Q in which q is a easy adjustable quantity. So by this method, we have benefit that $\triangle\bar{k}_L$ be controllable.